American University of Beirut
Department of Electrical and Computer Engineering

EECE 440 Signals and Systems

Homework 4 – Solution

Problem 1

Signal at b

$$b(t) = m_1(t) + 2m_2(t)\cos(10{,}000t)$$

Signal at c

$$c(t) = 2m_1(t)\cos(20{,}000t) + 2m_2(t)\cos(10{,}000t) + 2m_2(t)\cos(30{,}000t)$$

C(t) is a band-pass signal that rum from 9500 rad/s to 30500 rad/s. Therefore, its bandwidth is 21000 rad/s.

Problem 2

Y(t) is a low pass signal of bandwidth 12 rad/s.

Problem 3

The signal at the input of the filter is:

$$x(t) = \frac{1}{2}\text{rect}\left(\frac{t}{T}\right)\cos\omega_0 t + \frac{1}{2}\text{rect}\left(\frac{t}{T}\right)\cos(3\omega_0 t)$$

The signal at the output of the filter is:

$$g(t) = \frac{1}{2}\text{rect}\left(\frac{t}{T}\right)\cos\omega_0 t = \frac{1}{2}f(t)$$

Problem 4

$$|H(\omega)| = \frac{\omega}{\sqrt{1+\omega^2}}$$

Plotting $|H(\omega)|$ vs ω, we can easily conclude that the filter is a high-pass filter

Problem 5

At the input of the filter

$$S(\omega) = 12\pi[\delta(\omega-160) + \delta(\omega+160)] + 20\pi[\delta(\omega-220) + \delta(\omega+220)]$$

At the output of the filter

$$G(\omega) = 20\pi[\delta(\omega-220) + \delta(\omega+220)]$$
$$g(t) = 20\cos(220t$$

$$P_g = \frac{(20)^2}{2} = 200 \text{ Watts}$$

Problem 6

From your notes

$$g(t) = f(t)\cos(\omega_0 t) + \hat{f}(t)\sin(\omega_0 t)$$

Problem 7

The sinusoidal signal has a frequency of 1 Hz or a period of 1 s. The maximum allowable time between intervals should be T=$\frac{1}{2B} = \frac{1}{2}$ s.

Problem 8

Discussed in class

Problem 9

$$s(t) = 10\cos(2000\pi t) + 2.5\cos(2200\pi t) + 2.5\cos(1800\pi t) + 2.5\cos(2400\pi t) + 2.5\cos(1600\pi t)$$

b. Total power = $\frac{(10)^2}{2} + \frac{(2.5)^2}{2} + \frac{(2.5)^2}{2} + \frac{(2.5)^2}{2} + \frac{(2.5)^2}{2}$
= 50+2(6.25)=62.5 Watts

c. The total sideband power = total power – Carrier power = 62.5 -50
=12.5 Watts

d. Modulation index $= \mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$
Using MATLAB: A_{max} = 1, and A_{min}= -0.5
Therefore, $\mu = 0.64$

Problem 10

a. Carrier Power = $P_c = \frac{A_c^2}{2} = 50$ watts, implies that A_c=10 Volts

b. $\omega_c = 2\pi \times 10^6 \text{rad/s}$

c. $\mu = a_m k_a$ =0.8

d. $\omega_m = 2\pi \times 510^3 \text{rad/s}$

$$s(t) = 10[1 + 0.8\cos(10{,}000\pi t]\cos(2{,}000{,}000\pi t)$$

Problem 11

a. Modulation index $= \mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$
Using Plot: A_{max} = 12, and A_{min}= 4
Therefore, $\mu = 0.8$

b. $A_{max} = A_c[1 + \mu]$, implies that $A_c = 8\text{Volts}$

Problem 12

a. The AM wave

$$s(t) = A_c[1 + k_a m(t)]\cos(\omega_c t)$$

for m=0.5, $k_a = 1/16$

Therefore, the AM wave is written as:

$$s(t) = 100[1 - (6/16)\cos(20\pi t) - (2/16)\cos(60\pi t)]\cos 200\pi t$$

b. $s(t) = 100[1 + (1/16)m(t)]\cos(200\pi t) - A_c k_a \hat{m}(t)\sin(200\pi t)$